CUTOFF FREQUENCIES IN FIN LINES

CALCULATED WITH A TWO-DIMENSIONAL TLM-PROGRAM

Yi-Chi Shih, Wolfgang J.R. Hoefer,
Department of Electrical Engineering,
University of Ottawa,
Ottawa, Ontario, Canada,
K1N 6N5

A.E. Ros,
Ecole Nationale de
Travaux Publics de
Côte d'Ivoire,
Yamoussokro, Côte d'Ivoire

ABSTRACT

This paper presents the cutoff frequencies of selected insulated and unilateral fin line structures as calculated with a two-dimensional TLM-program. Through careful correction of truncation, velocity and coarseness errors associated with this method, an overall accuracy of 1% is achieved. This method therefore provides an excellent reference for verifying other existing methods.

INTRODUCTION

Various methods for evaluating the electrical parameters of fin lines have been presented by Meier [1] Hofmann [2], Saad and Begemann [3] Chang and Itoh [4] Hoefer [5] and Hoefer and Ros [6]. Since there is some disagreement between the results published by these authors, it is appropriate to verify them either by making very careful measurements or by using a numerical method inspiring the same confidence.

We have used a two-dimensional TLM (Transmission Line Matrix) program to evaluate the cutoff frequencies and the equivalent dielectric constant at cutoff for insulated and unilateral fin lines. Errors associated with the TLM-method have been carefully corrected to ensure that these values are accurate within ± 1%. The results obtained are therefore an excellent reference for the verification of other methods.

FEATURES OF THE TLM-PROGRAM

A shunt-connected transmission line matrix [8] was used to simulate transverse wave propagation in insulated and unilateral fin lines as shown in Fig. 1. Therefore, boundary conditions in the TLM-model are dual to those in the real structure, i.e. magnetic walls in the model correspond to electric walls in the fin line and vice versa. The impulse response of the network yields, after Fourier transformation, the spectrum of eigenmodes in the structure.

A typical program for a unilateral fin line simulated by a 34 x 10 TLM-matrix requires a core memory of 17 K bytes. Programs of this size can easily be implemented on a PDP-11/34 computer. When 1200 iterations are executed on an IBM 360, the required CPU-time is about 80 seconds.

ERROR CORRECTION

The three principal errors affecting the TLM-solution are the truncation, velocity and coarseness errors.

Truncation error stems from the necessity to limit the impulse response of the TLM-network in time. The maximum truncation error is given by the following expression [8]:

$$E_T = \frac{\Delta S}{\Delta \ell/\lambda_c} = \pm \frac{3\lambda_c}{S N^2 \pi^2 \Delta \ell} \tag{1}$$

λ_c is the cutoff wavelength to be calculated, and S is the frequency separation (expressed in terms of $\Delta\ell/\lambda$) between two neighboring peaks in the frequency response obtained from the impulse response via Fourier transform. N is the number of iterations processed. $\Delta\ell$ is the mesh parameter of the transmission line matrix, and λ is the free space wavelength. Normally, the truncation error is smaller than the maximum value given by eqn. (1). It decreases rapidly with an increasing number of iterations.

Velocity error arises if the velocity of pulse propagation in the TLM-network is assumed to be the same in all directions and equal to $v = c/\sqrt{2}$ (or $v' = c/\sqrt{2\varepsilon_r}$ in the dielectric substrate). However, in the case of the distorted TE_{n0}-modes ($n = 1,2,3,\ldots$) in fin lines, propagation occurs essentially in the direction parallel to the broad waveguide wall. The velocity in the TLM-network is, for that particular direction, given by the following transcendental equations;

in air: $$\sin\left(\frac{\beta_n \Delta\ell}{2}\right) = \sqrt{2}\,\sin\left(\frac{\omega\Delta\ell}{2c}\right) \tag{2}$$

in the dielectric: $$\sin\left(\frac{\beta'_n \Delta\ell}{2}\right) = \sqrt{2\varepsilon_r}\,\sin\left(\frac{\omega\Delta\ell}{2c}\right) \tag{3}$$

where β_n and β'_n are the wavenumbers in air and dielectric respectively. When the velocity of transverse propagation is determined using eqns. (2) and (3), the velocity error is typically smaller than 0.2% [7].

Coarseness error is due to the fact that in the vicinity of the fin edge, the TLM-network cannot resolve the highly nonuniform fields with infinite accuracy due to its finite mesh size. Obviously, this error can be made as small as one desires by choosing an ever finer mesh ($\Delta\ell \to 0$), but there are limits to this refinement. However, calculations show that the fundamental cutoff frequency, when calculated for networks with different mesh sizes, decreases practically linearly as $\Delta\ell/b$ increases (see Fig. 2). The coarseness error can thus be dramatically reduced by linear extrapolation toward $\Delta\ell/b = 0$. To this end, the cutoff frequency is extrapolated by fitting a line through three frequency values corresponding to three different mesh parameters $\Delta\ell$, as demonstrated in Fig. 2.

Overall error. In a recent study [7] we have evaluated the cutoff frequencies of finned waveguides (identical to fin lines with $\varepsilon_r=1$) using three completely different methods, namely the transverse resonance method, the finite element method and the TLM-method.

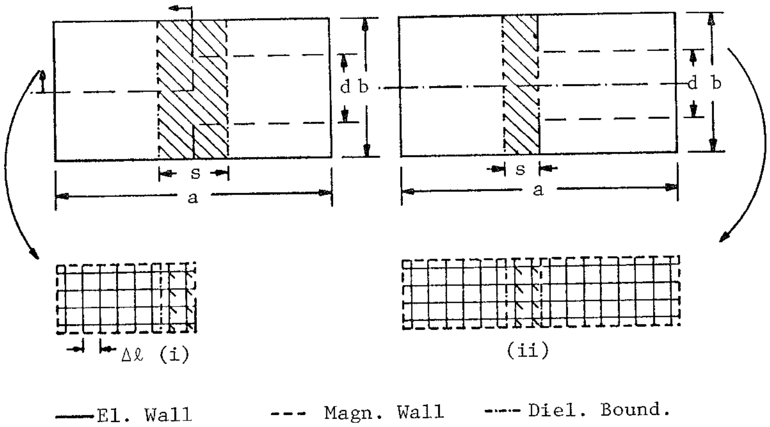

Fig. 1 Cross-sections of insulated (i) and unilateral (ii) fin lines. Below the sections, the simulating TLM-network is shown. For reasons of symmetry, only one quarter of the insulated and one half of the unilateral fin line section is modeled. In the TLM-models, boundaries are dual to those in the original structure since a shunt-connected transmission line matrix is used.

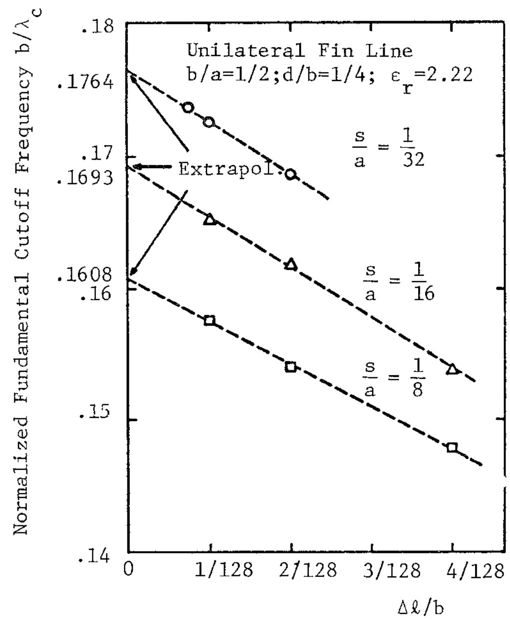

Fig. 2 Coarseness error is practically eliminated by linear extrapolation of cutoff frequency values towards $\Delta 1 = 0$.

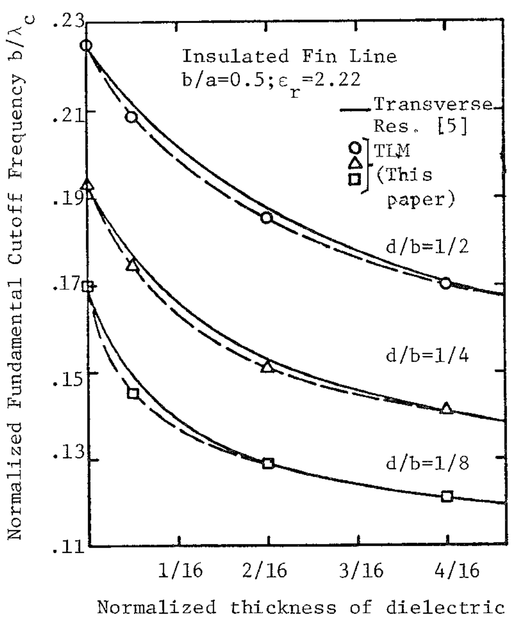

Figure 3 ←

Figure 4 →

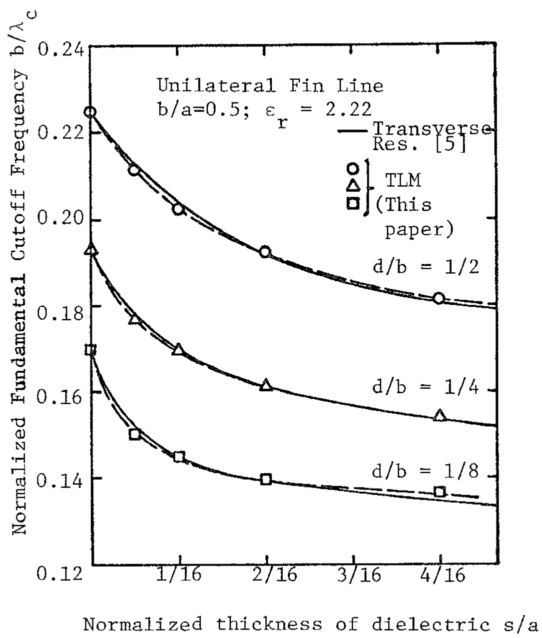

Fig. 3 and 4 Normalized fundamental cutoff frequency in fin lines. Results obtained with the Transverse Resonance Method [5] are compared with the TLM-solution.

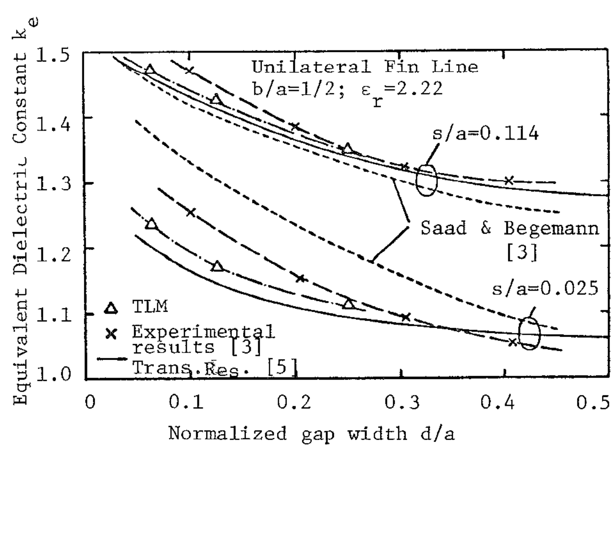

Fig. 5 Comparison of equivalent dielectric constant of unilateral fin line obtained with different methods.

For gap widths ranging from $d/b = 1/2$ to $1/16$, all three methods yield frequency values which agree within 0.5%. Since

i) there is no indication that all three methods are affected by a systematic error of identical sign and magnitude,

ii) the introduction of a dielectric with $\varepsilon_r > 1$ represents no new source of error in the TLM-calculation,

iii) the coarseness error (which is the major source of error) can be eliminated by the same linear extrapolation procedure as the one used for finned waveguide,

it is reasonable to assume that the TLM-method, when applied to fin line analysis, yields cutoff frequencies which are accurate within at least ± 1%, for all gap widths of practical interest.

RESULTS

Figures 3 and 4 show the normalized fundamental cutoff frequencies (b/λ_c) of insulated and unilateral fin lines respectively. In both cases, the waveguide aspect ratio is $b/a = 0.5$, and the substrate permittivity is $\varepsilon_r = 2.22$. The three curves are obtained for normalized gap widths $d/b = 1/2$, $1/4$ and $1/8$. For comparison, results obtained with the transverse resonance method [5] are presented in the same diagrams.

Fig. 5 compares the equivalent dielectric constant k_e for unilateral fin lines at cutoff, obtained with the TLM-method, the transverse resonance method [5] and approximate results published by Saad and Begemann [3]. The equivalent dielectric constant is defined by

$$k_e = k_{c\,air}^2 / k_{cd}^2 \tag{4}$$

where k_{cd} is the cutoff wavenumber ($k_{cd} = 2\pi/\lambda_c$) in the fin line, and $k_{c\,air}$ is the cutoff wavenumber for the same structure if the relative permittivity of the dielectric is unity(air).

Finally, some results obtained with a special finite element program for inhomogeneously filled waveguides (to be published) are compared with TLM-results in Table 1. Results agree within 1%.

d/b	b/λ_c TLM-Method	b/λ_c Finite El. Method	Difference in %
1/8	0.1285	0.1299	1%
1/4	0.1512	0.1529	1%
1/2	0.1853	0.1868	0.8%

Table 1. Normalized fundamental cutoff frequencies b/λ_c for insulated fin line obtained with the TLM and finite element methods. $b/a = 1/2$; $s/a = 1/8$; $\varepsilon_r = 2.22$ (see Fig. 1 for definition of dimensions)

CONCLUSION

The two-dimensional TLM-method provides an excellent reference for verifying other methods to evaluate the cutoff frequencies of fin lines. An accuracy of ± 1% is achieved by correcting the velocity and coarseness errors associated with the TLM-solution, and by using a sufficient number of iterations in the calculation of the impulse response to keep the truncation error negligeably small. The comparison between other methods [3], [5] and the TLM-solution confirm that the former are sufficiently accurate for most practical applications. Hoefer's method [5] appears to be generally more accurate than the approximate solution by Saad and Begemann [3].

REFERENCES

[1] P.J. Meier: "Integrated Fin Line Millimeter Components, IEEE-Trans. on Microwave Theory and Techniques, vol. MTT-22, No. 12, Dec. 1974, pp.1209-16

[2] H. Hofmann: "Dimension of Planar Waveguides for Millimeter-Wave Application", Archiv. F. Elektron. u. Übertrag.-Techn., Vol. 31, No. 1, Jan. 1977, pp. 40-44.

[3] A.M.K. Saad, G. Begemann: "Electrical Performance of Fin Lines of Various Configurations", Microwaves, Optics and Acoustics, Vol. 1, No. 2, Jan. 1977, pp. 81-88.

[4] C. Chang, T. Itoh: "Spectral Domain Analysis of Dominant and Higher Order Modes in Fin Lines", 1979 IEEE-MTT-S Intl. Microwave Symposium, April 28 - May 2, Orlando, Florida, U.S.A.

[5] W.J.R. Hoefer: "Fin Line Design Made Easy", 1978 IEEE-MTT-S Intl. Microwave Symposium, June 27-29, Ottawa, Ontario, Canada.

[6] W.J.R. Hoefer, A. Ros: "Fin-Line Parameters Calculated with the TLM-Method", 1979 IEEE-MTT-S Intl. Microwave Symposium, April 28- May 2, Orlando, Florida, U.S.A.

[7] Y.-C. Shih, W.J.R. Hoefer: "The Accuracy of TLM-Analysis of Finned Rectangular Waveguides", to appear in the IEEE-Trans. on Microwave Theory and Techniques.

[8] P.B. Johns:"Application of Transmission Line Matrix Method to Homogeneous Waveguides of Arbitrary Cross-Section", Proc. IEEE, Vol. 119,No. 8, August 197

ACKNOWLEDGEMENT

This work was supported by the Natural Sciences and Engineering Research Council of Canada under grant no. A7620 and by the Department of Communications of the Government of Canada.